MOMENTUM PRINCIPLE

1. Momentum Equation Derivation

Forces acting on a single particle by using Newton second law is equal to

$$\sum F = ma \qquad \sum F = \frac{d(mv)}{dt}$$

For a system contains a group of particles, the above equation

$$\sum F = \frac{d(Mom)_{sys}}{dt}$$

Where: $\frac{d(Mom)_{sys}}{dt}$ denotes the total momentum of all masses forming the system

It is known from Ch. (5) that the Reynolds transport theorem
For the Mass:

$$\frac{d(Mass)_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho dQ + \int_{CS} \rho V \bullet dA$$

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$$\frac{d(Mass)_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho dQ + \int_{CS} \rho V \bullet dA$$

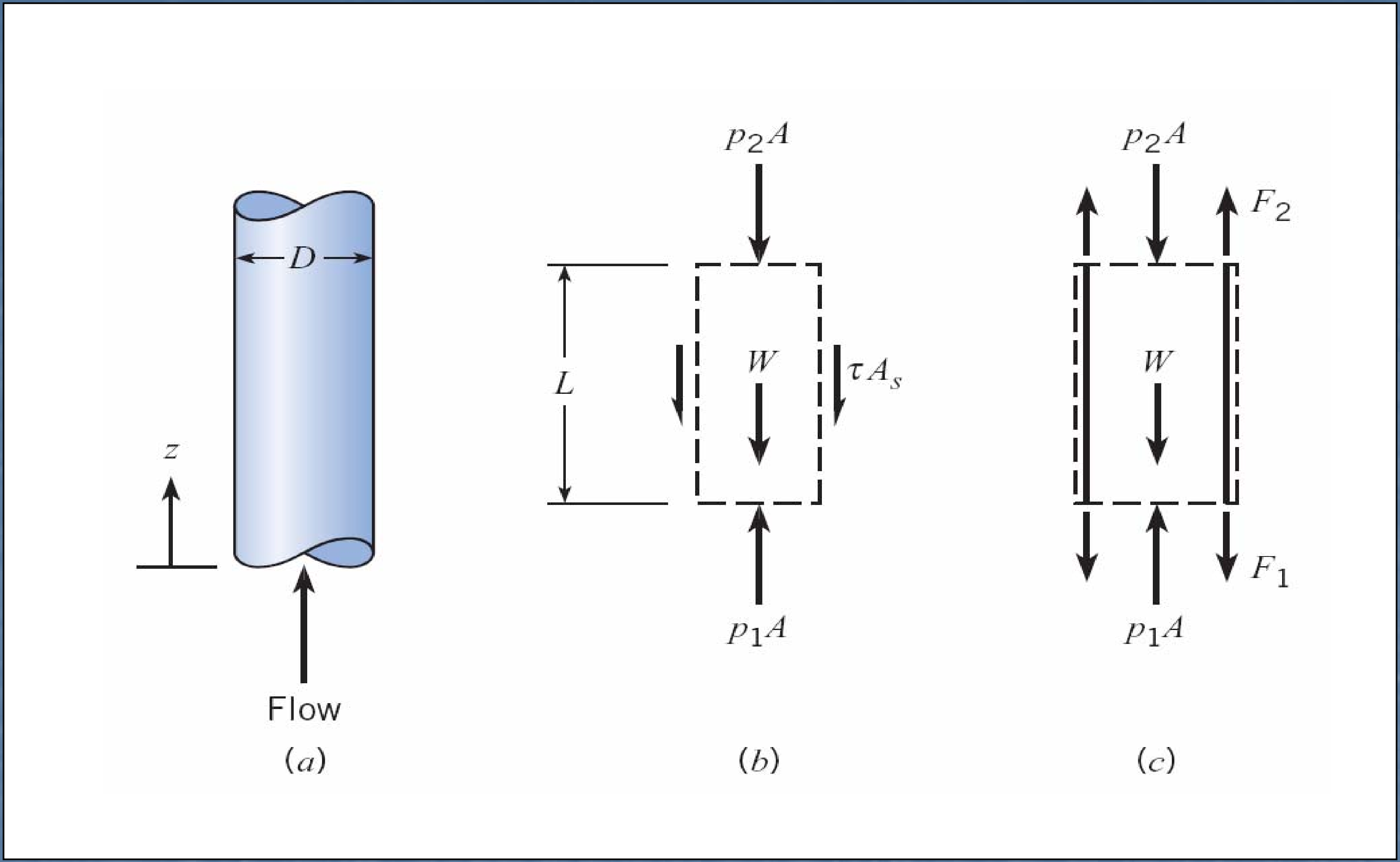
Multiply the above equation by velocity (V), we have

$$\frac{d(Mom)_{sys}}{dt} = \frac{d}{dt}\int_{cv} v\rho dQ + \int_{cs} v\rho V \bullet dA$$

$$\sum F = \frac{d}{dt}\int_{CV} (\rho Q)V + \sum_{CS} (\dot{m}V)_{out} - \sum_{CS} (\dot{m}V)_{in}$$

The momentum principle for a control surface

Interpretation of Momentum Equation



Forces associated with flow in a pipe: **(a)** pipe schematic, **(b)** control volume situated inside the pipe, and **(c)** control volume surrounding the pipe.

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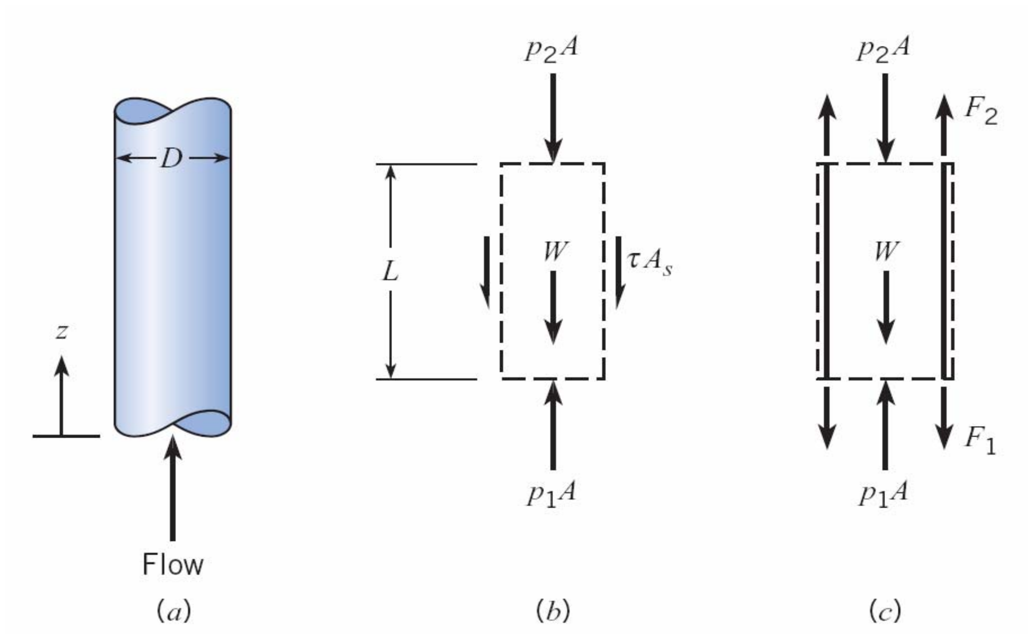
Case (b)

$$\sum F_z = \frac{\pi d^2}{4}(p_1 - p_2) - \tau\pi dL - \gamma\left(\frac{\pi d^2}{4}\right)L$$

Case (c)

$$\sum F_z = \frac{\pi d^2}{4}(p_1 - p_2) - F_1 + F_2 - \left(W_p + \gamma\frac{\pi d^2}{4}\right)L$$



(F_1, F_2)

represent forces due to pipe wall

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3. Momentum Accumulation

The momentum principle for a control surface is given by,

$$\sum F = \frac{d}{dt}\int_{cv} v\rho dQ + \sum_{CS}(\dot{m}v)_{out} - \sum_{CS}(\dot{m}v)_{in}$$

The momentum accumulation $= \frac{d}{dt}\int_{cv} v\rho dQ$

The momentum accumulation for a steady flow **= zero**

The momentum accumulation for a stationary structure **= zero**

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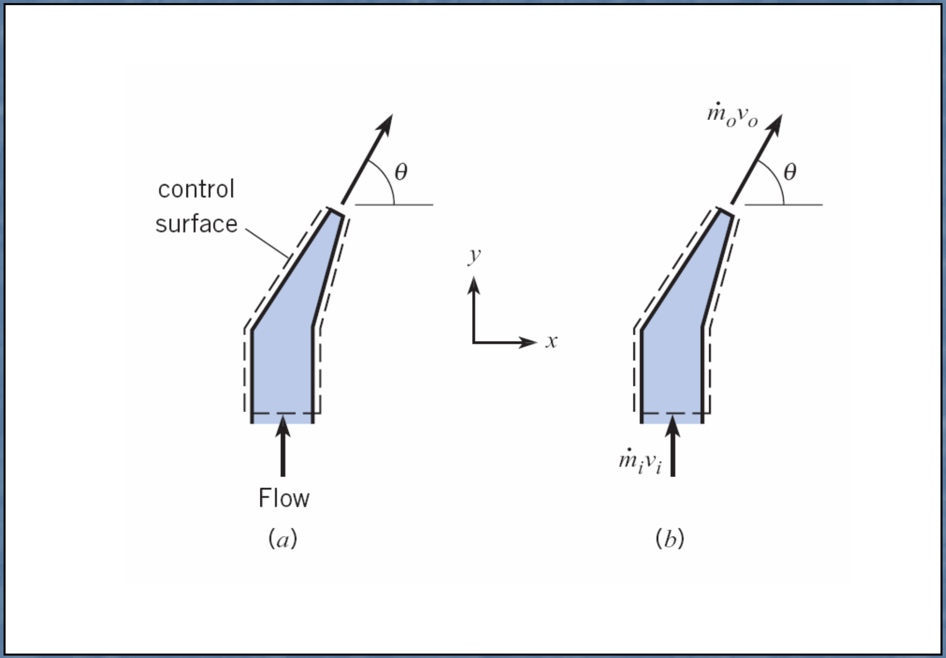
Momentum Diagramme



Momentum flow:

$$\sum_{CS}(\dot{m}v)_{out} - \sum_{CS}(\dot{m}v)_{in} = [\dot{m}v_{out}\cos\theta]i + [\dot{m}v_{out}\sin\theta - \dot{m}v_{in}]j$$

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The Momentum Equation for Cartesian Coordinates

X-direction:

$$\sum F_X = \frac{d}{dt}\int_{cv} v_X \rho dQ + \sum_{CS} (\dot{m}v)_{outX} - \sum_{CS} (\dot{m}v)_{inX}$$

Y-direction:

$$\sum F_Y = \frac{d}{dt}\int_{cv} v_Y \rho dQ + \sum_{CS} (\dot{m}v)_{outY} - \sum_{CS} (\dot{m}v)_{inY}$$

Z-direction:

$$\sum F_Z = \frac{d}{dt}\int_{cv} v_Z \rho dQ + \sum_{CS} (\dot{m}v)_{outZ} - \sum_{CS} (\dot{m}v)_{inZ}$$

END OF LECTURE (1)